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ERRATA: PRESENTATIONS OF THE MAPPING CLASS GROUP

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In [W] the second author gave a presentation for the mapping class group $M_{n,k}$ of an orientable surface $F_{n,k}$ of genus $n \geq 1$ with $k = 0$ or 1 boundary components. The statement of Theorem 2 in that paper contains an error. In this note we correct the error. At the same time we correct inaccuracies in [BW], which studied the same circle of ideas and made use of results from [W]. We are grateful to R. Piergallini for pointing out these errors to us and correcting some of them.

1. The statement of Theorem 2 in [W] is incorrect because the definition of d_n which is given in [W] is incorrect. In the definition of d_n one should set

$$v_1 = (b_2a_2b_1a_1^2b_1a_2b_2)^{-1}d(b_2a_2b_1a_1^2b_1a_2b_2).$$

With this definition d_n represents a Dehn twist with respect to the curve δ_n of Figure 5, as claimed.

Remark 1a: The proof of Theorem 2 in [W] does not use the formula for d_n, but it does use the fact that d_n represents a Dehn twist about δ_n. Therefore the proof of Theorem 2 is correct. In fact d_n can be replaced by any other product of the generators of $M_{n,1}$ which represents the Dehn twist on $F_{n,1}$ with respect to δ_n.

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Remark 1b: The presentation for $M_{n,1}$ which was given in §1 of [B] was based upon Theorem 2 of [W], but it did not use the incorret definition of d_n and it is correct.

2. The proof of Theorem 2 given on page 173 of [W] is not valid for n=1. In this case the relation $\omega = (a_1 b_1 a_1)^4 = 1$ is needed and it is not a consequence of the other relations. It must therefore be added to obtain a presentation of $M_{1,0} \cong SL(2; \mathrm{Z})$ with generators a_1, b_1. A full set of defining relations in this case is $a_1 b_1 a_1 = b_1 a_1 b_1$ and $(a_1 b_1 a_1)^4 = 1$. This was well-known long before [W] was published.

3. The symbol $\vee$ in relation C on line 2 of page 158 of [W] is a misprint. It should be replaced by the letter v, as in line 3 of page 158.

4. The statement of Lemma 2.5 of [BW] should be written in the following form: *Let x and y be nonliftable intervals which meet at one common end point. Then $z = (x)y$ is liftable $\Leftrightarrow z_1 = (x)y^2$ is not liftable $\Leftrightarrow z_2 = (x)\overline{y}$ is not liftable.* In fact, the lemma is used in this form in the proof of Lemmas 3.6 and 3.10.

5. In the statement of Lemma 3.6 of [BW] the formula $d_k = (u_k)\overline{v}_k x_k$ should be replaced with $d_k = (u_k)v_k$.

6. In the fourth line of the proof of Lemma 3.7 of [BW] the symbol α_1' should be replaced with the symbol α_i'.

7. On page 36 of [BW], after relation (ii), one should add the following statement:

* By relation (9) of [7] the following holds:

$$(d_4)x_5x_4x_3x_2^2x_3x_4x_5 = (d_4)\overline{x}_5\overline{x}_4\overline{x}_3\overline{x}_2^2\overline{x}_3\overline{x}_4\overline{x}_5.$$

Hence (ii) is equivalent to relation (B) in Theorem 1 of [7]

8. The definition of the braids t_1 and t_2 in [BW] should be changed as follows:

$$t_1 = (d_4)\overline{x}_5\overline{x}_4\overline{x}_6\overline{x}_5 \quad \text{and} \quad t_2 = (t_1)\overline{x}_3\overline{x}_2\overline{x}_4\overline{x}_3\ .$$

With these definitions the interval t_1 and t_2 coincide with the first two intervals in Figure 6.

9. The braid t_4, as defined on page 38 of [BW], does not coincide with the third interval in Figure 6, but it is equivalent to it. This can be seen as follows: we insert $(x_1^3)\overline{x}_2\overline{x}_3\overline{x}_4$ between x_5 and d_4 in the expression of t_4; then using (i) we get

$$t_3 = (d_4)\overline{x}_5\overline{x}_6\overline{x}_7x_3x_4x_5x_6x_2x_3x_4x_5\overline{x}_1^3\overline{x}_2\overline{x}_3\overline{x}_4d_4\overline{x}_5\overline{x}_6x_7 \ ,$$

which is the third interval of Figure 6.

10. The relator $d_{2g+1}\overline{x}_{2g+2}$ in the statement of Theorem 6.1 of [BW] should be replaced with $d_{2g}\overline{x}_{2g+2}$, where

$$d_{2g} = (x_{2g})x_{2g-1}x_{2g-2}\cdots x_2x_1^2x_2\cdots x_{2g-2}x_{2g-1}^2x_{2g-2}\cdots x_2x_1$$

and d_{2g+1} should be replaced with d_{2g} in the proof of Theorem 6.1.

d_{2g} lifts to δ_n of [7]; it is a product of generators different from x_{2g+2}, by Theorem 3.1, and by Remark 1a of this errata any such product can be used in the new relation: d_{2g} commutes with y.

11. Line 10 of page 40 of [BW] should be

$$(\overline{x}_1^3)\overline{x}_2\overline{x}_3\cdots\overline{x}_{2g+2}x_0,\overline{x}_0,$$

References

[B] J. S. Birman, *Mapping class groups of surfaces,* in *Braids,* Contemporary Mathematics **78** (1988), 13–43.

[BW] J. S. Birman and B. Wajnryb, *3-Fold branched coverings and the mapping class group of a surface,* in *Geometry and Topology,* Springer-Verlag Lecture Notes **1167** (1985), 24–43.

[W] B. Wajnryb, *A simple presentation of the mapping class group of an orientable surface,* Israel Journal of Mathematics **45** (1983), 157–174.